

# Molecular structures in charmonium spectrum: The $XYZ$ puzzle

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We study in the framework of a constituent quark model the possible contributions of molecular structures to the  $XYZ$  charmonium like states. We analyze simultaneously the  $c\bar{c}$  structures and the possible molecular components in a formalism which allows us to treat channels below and above thresholds. The only molecular state found in the  $1^{++}$  sector correspond to the  $X(3872)$ . Molecular resonances also appear with other quantum numbers. So, the so called  $Y(3940)$  and the  $X(3915)$  are suggested to be  $J^{PC} = 0^{++}$  charmonium states. In the  $J^{PC} = 1^{--}$  sector we also found significant contributions of the molecular structures which can affect the phenomenology.

PACS numbers: 12.39.Pn, 14.40.Pq, 13.75.Lb

Keywords: potential models, charmonium.

## I. INTRODUCTION.

In the last few years several charmonium like states were observed with similar masses near  $3.9\text{ GeV}$  but with quite different properties and in very different production processes. Altogether these states were called the  $XYZ$  states. A complete list of these new states can be found in Ref. [1]. Among them we will only comment on the confirmed states.

This new charmonium era started around 2003 when the Belle Collaboration discovered the lightest one, the  $X(3872)$ , in the exclusive decay  $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$  [2]. The mass of the state was measured to be  $3872.0 \pm 0.6\text{ MeV}$  very close to the  $M_{D^0} + M_{D^{*0}}$  threshold. The width was found to be very small  $\Gamma < 2.3\text{ MeV}$ . The state was soon confirmed by CDF [3], D0 [4] and BaBar [5]. By combination of the recent results reported by the Belle [6], BaBar [7] and CDF [8] Collaborations, the mass value is established at  $M_X = 3871.55 \pm 0.20\text{ MeV}$ .

An striking feature of the  $X(3872)$ , which cannot be explained by a simple  $c\bar{c}$  structure, is the ratio [9]

$$R_1 = \frac{X(3872) \rightarrow \pi^+ \pi^- \pi^0 J/\psi}{X(3872) \rightarrow \pi^+ \pi^- J/\psi} = 0.8 \pm 0.3. \quad (1)$$

The dipion mass spectrum in the  $\pi^+ \pi^- J/\psi$  channel shows that the pions come from the  $\rho^0$  resonance. On the other hand the  $\pi^+ \pi^- \pi^0$  mass spectrum has a strong peak around  $750\text{ MeV}$  suggesting that the process is dominated by an  $\omega$  meson. Although this number should be corrected by the strong phase suppression of the  $\omega J/\psi$  channel against the  $\rho J/\psi$  one, the ratio  $R_1 \sim 1$  is incompatible with a traditional charmonium assumption and is telling us that some isospin mixing is needed and that this mixing requires the contribution of both neutral and charged  $DD^*$  channels. Recently the Belle Collaboration [10] measured the ratio

$$R_2 = \frac{\Gamma(X(3872) \rightarrow \gamma \Psi(2S))}{\Gamma(X(3872) \rightarrow \gamma J/\psi)} \leq 2.1 (\text{at } 90\% \text{ C.L.}) \quad (2)$$

which complicates the interpretation of this state.

Two years later, the so called at that time  $Y(3940)$  was observed by the Belle Collaboration as a near-

threshold enhancement in the  $\omega J/\psi$  invariant mass distribution for the  $B \rightarrow K \omega J/\psi$  decay [11]. Belle reported a mass of  $M = 3943 \pm 11 \pm 13\text{ MeV}$  and a width  $\Gamma = 87 \pm 22 \pm 26\text{ MeV}$ . Belle observation seems to be confirmed by BaBar [12], although the mass ( $M = 3914 \pm 4.1\text{ MeV}$ ) and the width ( $\Gamma = 29 \pm 10\text{ MeV}$ ) were both smaller than Belle values. A later measurement of BaBar [13] confirmed this last mass value ( $M = 3919 \pm 3.8 \pm 2.0\text{ MeV}$ ). Very recently a new charmonium-like state, the  $X(3915)$ , has been reported by the Belle Collaboration in the  $\gamma\gamma \rightarrow J/\psi \omega$  decay [14]. The measured mass is  $M = 3914 \pm 3 \pm 2\text{ MeV}$  and the width  $\Gamma = 23 \pm 9\text{ MeV}$ . It has not yet been seen in the  $DD$  channel. Despite of the different mass and width of the first measurement of Belle, some authors [1, 15] subsumed both states under the name  $X(3915)$ , although the question if there is one or two different resonances is not definitely settled.

Another charmonium like state, the  $X(3940)$ , was observed in this region by Belle as a resonance in the double charmonium production  $e^+ e^- \rightarrow J/\psi DD^*$  in the mass spectrum recoil against the  $J/\psi$  [16]. Later on Belle confirmed the observation of  $X(3940) \rightarrow DD^*$  decay [17]. In addition Belle found a new charmonium like state  $X(4160)$  decaying into  $D^* D^*$ . Neither of them have been seen in the experimentally more accessible  $DD$  channel. As the decay of the  $Y(3940) \rightarrow DD^*$  was not observed in the  $B \rightarrow Y(3940) K$  [18], the  $X(3940)$  and the  $Y(3940)$  should be different states.

Two more states increase the experimental findings in this region. The  $Z(3930)$  was reported by Belle in the  $DD$  channel produced in  $\gamma\gamma$  collisions with mass and width  $M = 3929 \pm 6\text{ MeV}$  and  $\Gamma = 29 \pm 10\text{ MeV}$  [19].

Finally, it is worth to mention that one more resonance has been classified as 'well established' in Ref. [1]. It was found by the BaBar Collaboration [20] in the reaction  $e^+ e^- \rightarrow DD$  with a mass of  $3943 \pm 17 \pm 12\text{ MeV}$  and a width of  $52 \pm 8 \pm 7\text{ MeV}$  and confirmed by Belle [21]. It was called  $G(3900)$  in Ref. [20].

Concerning the quantum numbers of the new states, measurement of the angular correlations between final state particles in the  $X(3872) \rightarrow \pi^+ \pi^- J/\psi$  decay [22] together with small phase space available for the decay  $X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$  [23] strongly favors the  $J^{PC} = 1^{++}$

quantum numbers for this state. However some recent results seems to favor negative parity for this meson [13].

The situation is worse in the case of the  $X(3940)$ . It has not been seen in the  $DD$  channel which rules out the  $J^P = 0^{++}$  assignment. The dominant  $DD^*$  mode suggest that the  $X(3940)$  is the  $c\bar{c}(2^3P_1)$  state with  $J^{PC} = 1^{++}$  but this quantum numbers seems to be assigned to the  $X(3872)$ .

More consensus exists with the assignment of the  $Z(3930)$ . The  $DD$  decay mode makes it impossible to be the  $\eta_c(3S)$   $J^{PC} = 0^{-+}$  state. The two photon production can only produce  $DD$  states in  $0^{++}$  or  $2^{++}$  and these two cases can be distinguished looking to the  $dN/d\cos\theta$  distribution being  $\theta$  the angle between the incoming photon and the  $D$  meson in the  $\gamma\gamma$  center of mass system. This distribution is flat for  $0^{++}$  states and behaves like  $\sin^4\theta$  for  $2^{++}$ . The measurement of Belle [19] strongly favors the  $2^{++}$  case.

The  $J^{PC}$  assignment in the case that the  $Y(3940)$  was a  $c\bar{c}$  state is still unclear. A conventional  $c\bar{c}$  charmonium interpretation is in principle disfavored since it is well above the threshold for open charm decays and then these decay modes would dominate over the  $\phi J/\psi$  and  $\omega J/\psi$  decay rates. The  $X(3915)$  is preliminary assigned to be  $0^{++}$  or  $2^{++}$  [14].

Finally due to the entrance channel of the production reaction the  $G(3900)$  is clearly a  $J^{PC} = 1^{--}$  state.

Obviously it is difficult to accommodate all these states in a  $q\bar{q}$  scheme and all type of hypothesis about their structures (molecules, hybrids, tetraquarks) has been proposed in the literature (see [24] for a review).

The  $P = +$ ,  $C = +$  sector is specially suited for the coexistence of  $c\bar{c}$  states and molecular structures. The reason is the following: taken into account the negative intrinsic parity of the quark-antiquark pair, to get a positive parity state one needs at least one unit of angular momentum. However four quarks can reach the same positive parity with zero angular momentum. Then, the energy increase due to the angular momentum excitation may compensate the two additional light quark masses making the  $c\bar{c}$  and the  $c\bar{c}q\bar{q}$  structures almost degenerate.

This mechanism has been suggested in [25] as a possible explanation to the  $X(3872)$  properties. In this reference a coupled channel calculation of the  $1^{++}$  sector has been performed including  $c\bar{c}$  and  $DD^*$  molecular configurations. The  $X(3872)$  appears as a dynamically generated  $c\bar{c}$  and  $DD^*$  molecule by the coupling to a  $\chi_{c1}(2P)$  quark state. Although the  $c\bar{c}$  mixture is less than 10%, it is important to bind the molecule. In addition  $\pi^+\pi^- J/\psi$  decay modes data from Belle and BaBar are reasonably explained.

In this paper we will propose a theoretical explanation of the nature of some of these states using the same constituent quark model of Ref. [25]. This has been successfully used to describe hadronic spectroscopy and hadronic reactions [26–29] and is based on the assumption that constituent quark mass is a consequence of the spontaneous chiral symmetry breaking and has been recently

applied to the study of the energy spectrum and decay properties of the  $J^{PC} = 1^{--}$  charmonium sector [30]. Taken into account the existence of several thresholds in this energy region we develop a formalism which treats simultaneously molecular states above and below the different thresholds. This allows us to consider both dynamically generated molecular states by the coupling with  $c\bar{c}$  structures and molecular components in the dressed  $c\bar{c}$  states.

The paper is organized as follow. In Section II we first discuss the basic ingredients of the constituent model, the coupled channel formalism and the coupling mechanism between the different channels. Results and comments are given in Section III. Finally we summarize the main achievements of our calculation in Section IV.

## II. THE MODEL

### A. The constituent quark model

The constituent quark model used in this work has been extensively described elsewhere [28] and therefore we will only summarize here its most relevant aspects. The chiral symmetry of the original QCD Lagrangian appears spontaneously broken in nature and, as a consequence, light quarks acquire a dynamical mass. The simplest Lagrangian invariant under chiral rotations must therefore contain chiral fields, and can be expressed as

$$\mathcal{L} = \bar{\psi}(i\partial - M(q^2)U^{\gamma_5})\psi \quad (3)$$

where  $U^{\gamma_5} = e^{i\frac{\lambda_a}{f_\pi}\phi^a\gamma_5}$  is the Goldstone boson fields matrix and  $M(q^2)$  the dynamical (constituent) mass. This Lagrangian has been derived in Ref. [31] as the low-energy limit in the instanton liquid model. In this model the dynamical mass vanishes at large momenta and it is frozen at low momenta, for a value around 300 MeV. Similar results have also been obtained in lattice calculations [32]. To simulate this behavior we parametrize the dynamical mass as  $M(q^2) = m_q F(q^2)$ , where  $m_q \simeq 300$  MeV, and

$$F(q^2) = \left[ \frac{\Lambda_\chi^2}{\Lambda_\chi^2 + q^2} \right]^{\frac{1}{2}}. \quad (4)$$

The cut-off  $\Lambda_\chi$  fixes the chiral symmetry breaking scale.

The Goldstone boson field matrix  $U^{\gamma_5}$  can be expanded in terms of boson fields,

$$U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma_5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \dots \quad (5)$$

The first term of the expansion generates the constituent quark mass while the second gives rise to a one-boson exchange interaction between quarks. The main contribution of the third term comes from the two-pion exchange which has been simulated by means of a scalar exchange potential.

In the heavy quark sector chiral symmetry is explicitly broken and this type of interaction does not act. However it constrains the model parameters through the light meson phenomenology and provides a natural way to incorporate the pion exchange interaction in the open charm dynamics.

Below the chiral symmetry breaking scale quarks still interact through gluon exchanges described by the Lagrangian

$$\mathcal{L}_{gqq} = i\sqrt{4\pi\alpha_s} \bar{\psi}\gamma_\mu G_c^\mu \lambda_c \psi, \quad (6)$$

where  $\lambda_c$  are the SU(3) color generators and  $G_c^\mu$  the gluon field. The other QCD nonperturbative effect corresponds to confinement, which prevents from having colored hadrons. Such a term can be physically interpreted in a picture in which the quark and the antiquark are linked by a one-dimensional color flux-tube. The spontaneous creation of light-quark pairs may give rise at same scale to a breakup of the color flux-tube [33]. This can be translated into a screened potential [34] in such a way that the potential saturates at the same interquark distance

$$V_{CON}(\vec{r}_{ij}) = \{-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c). \quad (7)$$

Explicit expressions for these interactions are given in Ref. [35].

### B. The coupled channel calculation

In this section we present the formalism for the coupling of molecular structures with the  $c\bar{c}$  spectrum. We start defining the meson wave functions we will use all along the paper. To found the quark-antiquark bound states we solve the Schrödinger equation using the Gaussian Expansion Method [36]. In this method the radial wave functions solution of the Schrödinger equation are expanded in terms of basis functions

$$R_\alpha(r) = \sum_{n=1}^{n_{max}} b_n^\alpha \phi_{nl}^G(r) \quad (8)$$

where  $\alpha$  refers to the channel quantum numbers. The coefficients  $b_n^\alpha$  and the eigenenergy  $E$  are determined from the Rayleigh-Ritz variational principle

$$\sum_{n=1}^{n_{max}} \left[ (T_{n'n}^{\alpha'} - EN_{n'n}^{\alpha'}) b_n^{\alpha'} + \sum_{\alpha} V_{n'n}^{\alpha'\alpha} b_n^{\alpha} \right] = 0 \quad (9)$$

where the operators  $T_{n'n}^\alpha$  and  $N_{n'n}^\alpha$  are diagonal and the only operator which mix the different channels is the potential  $V_{n'n}^{\alpha\alpha'}$ .

A crucial problem of the variational methods is how to choose the radial functions  $\phi_{nl}^G(r)$  in order to have a minimal, but enough, number of basis functions. Following [36] we employ gaussian trial functions whose ranges are in geometric progression. The geometric progression

is useful in optimizing the ranges with a small number of free parameters. Moreover the distribution of the gaussian ranges in geometric progression is dense at small ranges, which is well suited for making the wave function correlate with short range potentials. The fast damping of the gaussian tail is not a real problem since we can choose the maximal range much longer than the hadronic size.

To model the  $c\bar{c}$  system we assume that the hadronic state is

$$|\Psi\rangle = \sum_{\alpha} c_{\alpha} |\psi_{\alpha}\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_A \phi_B \beta\rangle \quad (10)$$

where  $|\psi_{\alpha}\rangle$  are  $c\bar{c}$  eigenstates of the two body Hamiltonian,  $\phi_M$  are  $q\bar{q}$  eigenstates describing the  $A$  and  $B$  mesons,  $|\phi_A \phi_B \beta\rangle$  is the two meson state with  $\beta$  quantum numbers coupled to total  $J^{PC}$  quantum numbers and  $\chi_{\beta}(P)$  is the relative wave function between the two mesons in the molecule. When we solve the four body problem we also use the gaussian expansion of the  $q\bar{q}$  wave functions obtained from the solution of the two body problem. This procedure allows us to introduce in a variational way possible distortions of the two body wave function within the molecule. To derive the meson-meson interaction from the  $qq$  interaction we use the Resonating Group Method (RGM).

The coupling between the two sectors requires the creation of a light quark pair  $n\bar{n}$ . Similar to the strong decay process this coupling should be in principle driven by the same interquark Hamiltonian which determines the spectrum. However Ackleh *et al.* [37] have shown that the quark pair creation  ${}^3P_0$  model [38], gives similar results to the microscopic calculation. The model assumes that the pair creation Hamiltonian is

$$\mathcal{H} = g \int d^3x \bar{\psi}(x) \psi(x) \quad (11)$$

which in the non-relativistic reduction is equivalent to the transition operator [39]

$$\begin{aligned} \mathcal{T} = & -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(p+p') \\ & \times \left[ \mathcal{Y}_1 \left( \frac{p-p'}{2} \right) b_{\mu}^{\dagger}(p) d_{\nu}^{\dagger}(p') \right]^{C=1, I=0, S=1, J=0} \end{aligned} \quad (12)$$

where  $\mu$  ( $\nu$ ) are the quark (antiquark) quantum numbers and  $\gamma' = 2^{5/2} \pi^{1/2} \gamma$  with  $\gamma = \frac{g}{2m}$  is a dimensionless constant that gives the strength of the  $q\bar{q}$  pair creation from the vacuum. From this operator we define the transition potential  $h_{\beta\alpha}(P)$  within the  ${}^3P_0$  model as [40]

$$\langle \phi_{M_1} \phi_{M_2} \beta | \mathcal{T} | \psi_{\alpha} \rangle = P h_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{cm}) \quad (13)$$

where  $P$  is the relative momentum of the two meson state.

Adding the coupling with charmonium states we end-up with the coupled-channel equations

$$\begin{aligned}
c_\alpha M_\alpha + \sum_\beta \int h_{\alpha\beta}(P) \chi_\beta(P) P^2 dP &= E c_\alpha \\
\sum_\beta \int H_{\beta'\beta}(P', P) \chi_\beta(P) P^2 dP + \sum_\alpha h_{\beta'\alpha}(P') c_\alpha &= E \chi_{\beta'}(P')
\end{aligned} \tag{14}$$

where  $M_\alpha$  are the masses of the bare  $c\bar{c}$  mesons and  $H_{\beta'\beta}$  is the RGM Hamiltonian for the two meson states ob-

tained from the  $q\bar{q}$  interaction. Solving the coupling with the  $c\bar{c}$  states we arrive to an Schrödinger-type equation

$$\sum_\beta \int \left( H_{\beta'\beta}(P', P) + V_{\beta'\beta}^{eff}(P', P) \right) \chi_\beta(P) P^2 dP = E \chi_{\beta'}(P') \tag{15}$$

where

$$V_{\beta'\beta}^{eff}(P', P; E) = \sum_\alpha \frac{h_{\beta'\alpha}(P') h_{\alpha\beta}(P)}{E - M_\alpha}. \tag{16}$$

Our aim is to find molecular states above and below thresholds in the same formalism. However, above the threshold we will find complex eigenenergies, where the imaginary part is related to the width of such states.

In order to find the poles of the  $T$  matrix we must be in the correct Riemann sheet, so we have to analytically continue all the potentials for complex momenta. Once the analytical continuation is performed, the previous coupled channel equations can be solved through the  $T(\vec{p}, \vec{p}', E)$  matrix, solution of the Lippmann-Schwinger equation,

$$T^{\beta'\beta}(P', P; E) = V_T^{\beta'\beta}(P', P; E) + \sum_{\beta''} \int V_T^{\beta'\beta''}(P', P''; E) \frac{1}{E - E_{\beta''}(P'')} T^{\beta''\beta}(P'', P; E) P''^2 dP'' \tag{17}$$

where  $V_T^{\beta'\beta}(P', P; E) = V^{\beta'\beta}(P', P) + V_{\beta'\beta}^{eff}(P', P; E)$ ,  $V^{\beta'\beta}(P', P)$  is the RGM potential and  $V_{\beta'\beta}^{eff}(P', P; E)$  is the effective potential due to the coupling to intermediate  $c\bar{c}$  states.

In this way we study the influence of the  $c\bar{c}$  states on the dynamics of the two meson states. This is a different point of view from the usually found in the literature

where the influence of two meson states (in general without meson-meson interaction) in the mass and width of  $c\bar{c}$  states is studied [40]. Our approach allows to generate new states through the meson-meson interaction due to the coupling with  $c\bar{c}$  states and to the underlying  $q\bar{q}$  interaction.

The  $T$  matrix of Eq. (17) can be factorized as [41]

$$T^{\beta'\beta}(P', P; E) = T_V^{\beta'\beta}(P', P; E) + \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(P'; E) \Delta_{\alpha'\alpha}^{-1}(E) \phi^{\alpha\beta}(P; E) \tag{18}$$

with  $\Delta_{\alpha'\alpha}^{-1}(E) = \left( (E - M_\alpha) \delta^{\alpha'\alpha} + \mathcal{G}^{\alpha'\alpha}(E) \right)^{-1}$  being the propagator of the mixed state and  $T_V^{\beta'\beta}(P', P; E)$  the

$T$  matrix of the RGM potential excluding the coupling to the  $c\bar{c}$  pairs.

The new functions  $\phi^{\beta\alpha}(P; E)$  can be interpreted as the

dressed  ${}^3P_0$  vertex by the RGM meson-meson interaction and are defined as

$$\phi^{\beta\alpha}(P; E) = h_{\beta\alpha}(P) - \sum_{\beta'} \int \frac{T_V^{\beta\beta'}(P, q; E) h_{\beta'\alpha}(q)}{q^2/2\mu - E} q^2 dq. \quad (19)$$

Resonances will appear as poles of the  $T$  matrix, namely as zeros of the inverse propagator of the mixed state. Therefore the resonance parameters are solutions of the equation

$$|\Delta_{\alpha'\alpha}(\bar{E})| = |(\bar{E} - M_\alpha)\delta^{\alpha'\alpha} + \mathcal{G}^{\alpha'\alpha}(\bar{E})| = 0 \quad (20)$$

with  $\bar{E}$  the pole position. This equation is solved by the Broyden method [42].

From the solution of (20) we obtain the energy and the total width of the resonance. However, we are faced with the problem of the definition of the partial width. A similar problem arise when one try to define the mass and the width of an unstable particle in a gauge independent way [43]. Let assume the case of a  $c\bar{c}$  bound state coupled to two meson states. If the two mesons are below threshold we get a mass shift of the particle mass but if they are above threshold the mass is also renormalized by the coupling and now the particle becomes unstable and ac-

quires a width. The conventional definition of mass and width are in this case.

$$\begin{aligned} M &= M_0 - \Re(\mathcal{G}(M)), \\ \Gamma &= 2 \frac{\Im(\mathcal{G}(M))}{1 + \Re(\mathcal{G}(M))} \end{aligned} \quad (21)$$

where  $M_0$  is the bare mass and  $\mathcal{G}(E)$  is the two meson loop. The partial width is defined by decomposing the numerator of the width in Eq. (21) into a sum of contributions of different two meson channels. However this cannot be done in our case because we use the complex value of the energy at the pole position to define the mass and width of the state

$$\bar{E} = M_0 - \mathcal{G}(\bar{E}). \quad (22)$$

With the usual parametrization  $\bar{E} = M_r - i\Gamma_r/2$  we obtain

$$\Gamma_r = 2\Im(\mathcal{G}(\bar{E})) \quad (23)$$

and we cannot follow the usual procedure to define the partial widths. Instead, following Ref. [43], we start from the  $S$ -matrix for an arbitrary number of  $c\bar{c}$  states

$$S^{\beta'\beta}(E) = S_{bg}^{\beta'\beta}(E) - i2\pi\delta^4(P_f - P_i) \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(k; E) \Delta_{\alpha'\alpha}(E)^{-1} \phi^{\alpha\beta}(k; E) \quad (24)$$

where  $k$  is the on-shell momentum of the two meson state and the propagator is

$$\Delta^{\alpha'\alpha}(E) = \left\{ (E - M_\alpha)\delta^{\alpha'\alpha} + \mathcal{G}^{\alpha'\alpha}(E) \right\} \quad (25)$$

Expanding around the pole, which now is defined as  $|\Delta(\bar{E})| = 0$ :

$$\begin{aligned} \Delta^{\alpha'\alpha}(E) - \Delta^{\alpha'\alpha}(\bar{E}) &= (E - \bar{E}) \left[ \delta^{\alpha'\alpha} + \mathcal{G}'^{\alpha'\alpha}(\bar{E}) \right] \\ &= (E - \bar{E}) \mathcal{Z}^{\alpha'\alpha}(\bar{E}) \end{aligned} \quad (26)$$

with

$$\mathcal{G}'^{\alpha'\alpha}(\bar{E}) = \lim_{E \rightarrow \bar{E}} \frac{\mathcal{G}^{\alpha'\alpha}(E) - \mathcal{G}^{\alpha'\alpha}(\bar{E})}{E - \bar{E}} \quad (27)$$

Then the  $S$ -matrix can be approximated in the neighborhood of the pole as

$$S^{\beta'\beta}(E) = S_{bg}^{\beta'\beta}(E) - i2\pi\delta^4(P_f - P_i) \sum_{\alpha, \alpha'} \phi^{\beta'\alpha'}(\bar{k}; \bar{E}) \frac{\mathcal{Z}_{\alpha'\alpha}(\bar{E})^{-1}}{E - \bar{E}} \phi^{\alpha\beta}(\bar{k}; \bar{E}) \quad (28)$$

assuming that

$$\mathcal{Z}_{\alpha'\alpha}(\bar{E}) = \sum_{\lambda} \mathcal{Z}_{\alpha'\lambda}^{1/2} \mathcal{Z}_{\lambda\alpha}^{1/2} \quad (29)$$

the  $S$ -matrix can be finally written as

$$S^{\beta'\beta}(E) = S_{bg}^{\beta'\beta}(E) - i2\pi\delta^4(P_f - P_i) \sum_{\alpha,\alpha',\lambda} \left[ \phi^{\beta'\alpha'}(\vec{k}; \vec{E}) \mathcal{Z}_{\alpha'\lambda}(E)^{-1/2} \right] \frac{1}{E - \bar{E}} \left[ \mathcal{Z}_{\lambda\alpha}(E)^{-1/2} \phi^{\alpha\beta}(\vec{k}; \vec{E}) \right]. \quad (30)$$

The vertex we are interested in is

$$S(X_c \rightarrow f)^{\beta\alpha} = \sum_{\lambda} \phi^{\beta\lambda}(\vec{k}; \vec{E}) \mathcal{Z}_{\lambda\alpha}(\vec{E})^{-1/2} \quad (31)$$

and the partial width can be defined as

$$\hat{\Gamma}_f = \int d\Phi_f |S(X_c \rightarrow f)|^2 \quad (32)$$

where the integral is over the phase space of the final state with  $(\sum_n p_n)^2 = M_r^2$ .

In the case of a two meson decay  $\hat{\Gamma}_\beta$  can be written as

$$\hat{\Gamma}_\beta = 2\pi \frac{E_1 E_2}{M_r} k_{0\beta} \sum_{\alpha', \alpha, \lambda} \phi^{*\beta\alpha'}(\vec{k}) \mathcal{Z}_{\alpha'\lambda}^*(\vec{E})^{-1/2} \mathcal{Z}_{\lambda\alpha}(\vec{E})^{-1/2} \phi^{\alpha\beta}(\vec{k}) \quad (33)$$

where  $k_{0\beta}$  is the onshell momentum of the two meson state.

Eq. (33) does not guarantee that the sum of the partial widths must be equal to the total width. In fact it is expected that  $\sum_f \hat{\Gamma}_f \neq \Gamma_r$ . To solve this problem we define the *branching ratios* by [43]

$$\mathcal{B}_f = \frac{\hat{\Gamma}_f}{\sum_f \hat{\Gamma}_f} \quad (34)$$

and the partial widths by

$$\Gamma_f = \mathcal{B}_f \Gamma_r. \quad (35)$$

### III. CALCULATIONS, RESULTS AND DISCUSSIONS

Using the formalism developed in section II we have performed a coupled channel calculation of molecular states and  $q\bar{q}$  pairs in different sectors of the charmonium spectrum. We use the parametrization of the  $q\bar{q}$  interaction, together with the values for quark masses and the strength of the  $^3P_0$  model of Ref. [44].

We have analyzed the positive parity sectors from 3.8 to 4.0 GeV where more of the new charmonium-like states appear. We have also calculate the effects of molecular structures for the  $J^{PC} = 1^{--}$  states in the region around 4.0 GeV.

#### A. Positive parity sector

In Table I we summarized the  $XYZ$  candidates below 4.0 GeV that we will discuss in this work together with

State	M (MeV)	$\Gamma$ (MeV)	$J^{PC}$	Decay mode
$X(3872)$	$3871.4 \pm 0.6$	$< 2.3$	$1^{++}$	$DD^*$
$X(3915)$	$3914 \pm 3 \pm 2$	$23 \pm 9$	?	$D^* D^*$
$Z(3930)$	$3929 \pm 5$	$29 \pm 10$	$2^{++}$	$DD$
$X(3940)$	$3942 \pm 9$	$37 \pm 17$	$1^{++}$	$DD^*$
$Y(3940)$	$3943 \pm 17$	$87 \pm 34$	? $C = +$	$J/\psi\omega$

Table I: Summary of candidates  $XYZ$  mesons discussed in this work.

State	M (MeV)	$J^{PC}$
$\chi_{c0}(2P)$	3909	$0^{++}$
$h_c(2P)$	3955	$1^{+-}$
$\chi_{c1}(2P)$	3947	$1^{++}$
$\chi_{c2}(2P)$	3968	$2^{++}$

Table II: Prediction from our  $c\bar{c}$  model in the region around 3970 MeV.

the most likely  $J^{PC}$  assignments and its dominant decay modes. In Table II we show the prediction in this region from the model of Ref. [44].

Although the model predicts four states in this region, the  $h_c(2P)$  has negative  $C$ -parity and does not match with the data we are looking for. Then we have only three states predicted by the quark model whereas experimentally one found 5 states.

As far as masses are concern, there is one clear identification: the  $\chi_{c2}$  match the  $Z(3930)$  mass (see Ref. [45] for more properties). For the rest of states one can briefly comment that the  $\chi_{c1}$  is too high in mass to be  $X(3872)$ . On the other hand the  $X(3940)$  cannot be a  $J^P = 0^{++}$  state because, while a clear signal for  $X(3940) \rightarrow DD^*$  is seen, there is no evidence for the  $X(3940)$  in either  $DD$  or  $\omega J/\psi$  decay channels. Then the most likely candidate for  $X(3940)$  is  $J^{PC} = 1^{++}$  with  $M = 3947$  MeV. Concerning the  $X(3915)$ , although it has not been seen in the  $DD$  channel due basically to its small branching ratio for this channel, it is a good candidate for our  $\chi_{c0}$  state.

With this preliminary assignments our  $c\bar{c}$  model predicts no candidates for the  $X(3872)$  and the  $Y(3940)$  if it finally exists.

The existence of two  $J^{PC} = 1^{++}$  almost degenerated in mass, namely the  $X(3940)$  and the  $X(3872)$  suggest that the  $1^{++}$   $c\bar{c}$  sector at these energies is more complicated than a simple  $c\bar{c}$  structure. Moreover, its extremely low binding energy makes the  $X(3872)$  an ideal candidate to a  $DD^*$  molecule.

$\gamma$	$E_{bind}$	$c\bar{c}(2^3P_1)$	$D^0D^{*0}$	$D^\pm D^{*\mp}$	$J/\psi\rho$	$J/\psi\omega$
0.231	-0.60	12.40	39.24	7.46	0.49	0.40
0.226	-0.25	8.00	86.61	4.58	0.53	0.29

Table III: Binding energy (in MeV) and channel probabilities (in %) for the  $X(3872)$  states for two different values of the  $\gamma$  parameter in the  $^3P_0$  model.

In a earlier publication [25] we performed a coupled channel calculation including the  $c\bar{c}(2^3P_1)$  pair together with the neutral and charged  $DD^*$  channels. In this work we found a bound state with an important molecular component. However one can wonder if the same effects can be found coupling the  $\rho J/\psi$  and  $\omega J/\psi$  neglected in this previous calculation.

To elucidate this point we have performed a full calculation including the two quark  $c\bar{c}(2^3P_1)$  together with the  $D^0D^{*0}$ ,  $D^\pm D^{*\mp}$ ,  $\rho J/\psi$  and  $\omega J/\psi$  channels. The coupling of the  $DD^*$  with the  $\rho J/\psi$  and  $\omega J/\psi$  channels is not enough to bind the molecule and it is mandatory to couple the  $c\bar{c}$  pair to reach a molecular bound state.

One striking feature of the  $X(3872)$  decays is the value of the ratio between the  $X(3872) \rightarrow \rho J/\psi$  and the  $X(3872) \rightarrow \omega J/\psi$  decay channels. As stated above this ratio suggests that some isospin mixing is needed to reproduce the experimental data. To introduce the isospin breaking in our calculation we will work in the charge basis instead of in the isospin symmetric one, allowing the dynamics of the system to choose the weight of the different components. Isospin is explicitly broken by the experimental meson masses.

To have an idea of the sensitivity of the  $X(3872)$  structure with the binding energy and having in mind that this ranges from 0.6 MeV to 0.25 MeV, we have fine-tuned the  $^3P_0$  gamma parameter to get exactly this two energies. We can see the results in Table III.

From this table one can see that the  $X(3872)$  is predominately a  $D^0D^{*0}$  molecule with a small admixture (less than 10%) of the  $c\bar{c}(2^3P_1)$  state and the charged  $D^\pm D^{*\mp}$  component. The two channels  $\rho J/\psi$  and  $\omega J/\psi$ , although important for the decays, are not significant with respect to the binding energy.

The influence of the different components can be determine from the  $X(3872)$  decays. The ratio

$$R_2 = \frac{\Gamma(X(3872) \rightarrow \gamma\Psi(2S))}{\Gamma(X(3872) \rightarrow \gamma J/\psi)} \quad (36)$$

is sensitive to the  $c\bar{c}(2^3P_1)$  component because in the molecular picture the radiative decay  $\Gamma(X(3872) \rightarrow \gamma\Psi(2S))$  is suppressed [46]. The first evidence of this decay was reported by BaBar with a branching fraction  $\frac{\Gamma(X(3872) \rightarrow \gamma\Psi(2S))}{\Gamma(X(3872) \rightarrow \gamma J/\psi)} = 3.4 \pm 1.4$  [47] which suggested a rather large value of the  $c\bar{c}(2^3P_1)$  component. However, in 2010, using a larger sample of the  $B \rightarrow X(3872)K$  decay, the Belle Collaboration has not found evidences for the radiative decay  $\Gamma(X(3872) \rightarrow \gamma\Psi(2S))$  giving the

$E_{bind}$	$\Gamma_{\gamma J/\psi}$	$\Gamma_{\gamma\Psi(2S)}$	$R_2$
-0.60	8.15	9.80	1.20
-0.25	5.25	6.31	1.20

Table IV: Decay widths (in keV) of the the  $X(3872)$  states and its ratio for two different values of the  $\gamma$  parameter in the  $^3P_0$  model.

$E_{bind}$	$\Gamma_{\pi^+\pi^-J/\psi}$	$\Gamma_{\pi^+\pi^-\pi^0J/\psi}$	$R_1$
-0.60	27.61	14.40	0.52
-0.25	24.18	10.64	0.44

Table V: Strong decay widths (in keV) of the the  $X(3872)$  states and its ratio for two different values of the  $\gamma$  parameter in the  $^3P_0$  model.

$Mass$	$c\bar{c}(2^3P_1)$	$D^0D^{*0}$	$D^\pm D^{*\mp}$	$J/\psi\rho$	$J/\psi\omega$
3871.5	8.00	86.61	4.58	0.53	0.29
3941.8	61.09	18.53	16.85	0.01	3.52

Table VI: Masses (in MeV) and channel probabilities (in %) for the  $X(3872)$  and  $X(3940)$  states.

upper limit  $\mathcal{B.R.} < 2.1$  at 90% C.L. [10]. In Table IV we show the width and the ratio for this two decays using the standard expression for the electric dipole transition [1]. One can see that the value of our  $c\bar{c}$  component is small enough to accommodate at the experimental results.

In Table V we show the calculated width for the decays  $X(3872) \rightarrow \pi^+\pi^-J/\psi$  and  $X(3872) \rightarrow \pi^+\pi^-\pi^0J/\psi$ . The result for the ratio is not far from the experimental value  $R_1 = 0.8 \pm 0.3$  [9]. Although the absolute value of both decay widths varies with the  $X(3872)$  binding energy their ratio is less sensitive. It is worth to notice that to obtain this ratio (close to the experimental value) it is enough to have only less than 30% of  $I = 1$  component because, as explained in the introduction, the different phase spaces of the  $\rho J/\psi$  and the  $\omega J/\psi$  channels conspires with the  $DD^*$  charged components to reproduce the experimental result.

As stated above the formalism developed in Section II allows us to treat simultaneously bound states and resonances above and below the thresholds. When we look for resonances above the  $DD^*$  threshold, we found a resonance at  $M = 3941.8$  MeV and  $\Gamma = 89.9$  MeV, which can be identified with the  $X(3940)$ . The different components for this resonance together with the  $X(3872)$  for the value  $\gamma = 0.226$  are shown in Table VI.

As seen from Table VI the  $X(3940)$  has almost the same value of the neutral and charged  $DD^*$  components. Therefore the isospin breaking has almost completely disappear and the resonance is basically  $I = 0$ . This justifies that the coupling with the  $\rho J/\psi$  channel is almost negligible.

This resonance decay basically through the  $DD^*$  com-

state	Mass	$\Gamma$	$c\bar{c}(2^3P_0)$	$D^0\bar{D}^0$	$J/\psi\omega$	$D_s D_s$	$J/\psi\phi$
$X(3915)$	3896.5	4.10	34.22	46.67	9.42	9.67	0.03
$Y(3940)$	3970	189.3	57.27	35.32	0.15	5.72	1.54

Table VII: Mass and total width (in MeV) and channel probabilities (in %) for the  $X(3915)$  and  $Y(3949)$ .

ponent being the branching ratio for the different channels  $\mathcal{B.R.}(X(3872) \rightarrow DD^*) = 0.89$ ,  $\mathcal{B.R.}(X(3872) \rightarrow \omega J/\psi) = 0.1$  and  $\mathcal{B.R.}(X(3872) \rightarrow \rho J/\psi) = 3 \cdot 10^{-4}$ . In this way the puzzle between this two states with the same quantum numbers seems to be solved in a satisfactory way.

The situation in the  $J^{PC} = 0^{++}$  is also puzzling. Before the discovery of the  $X(3915)$  signal, it was supposed that the  $Y(3940)$  could be the  $J^{PC} = 1^{++}$  or  $J^{PC} = 0^{++}$  state. If we look to the Table II, our  $c\bar{c}$  model predicts a  $J^{PC} = 0^{++}$  state at 3909 MeV and so the  $Y(3940)$  seems to be too high. Despite the fact that one of the states is narrow (the  $X(3915)$ ) and the other is broad (the  $Y(3940)$ ), lately there has been a tendency to consider that both are the same state. We have performed the same calculation as before but for the  $J^{PC} = 0^{++}$  sector. We include the  $c\bar{c}(2^3P_0)$  channel together with the molecular channels  $DD$  (3736.05 MeV),  $\omega J/\psi$  (3879.56 MeV),  $D_s D_s$  (3936.97 MeV),  $\phi J/\psi$  (4116.0 MeV) where the channel thresholds are given between parenthesis. The results are shown in Table VII.

We find a narrow state which can be identified with the  $X(3915)$ . This resonance is basically a mixture of  $c\bar{c}$  and a  $DD$  molecular component. Moreover a second wide resonance appears at  $M = 3970$  MeV. This resonance can be the old  $Y(3940)$ , today disappeared from the Particle Data Group. Its dominant component is the  $c\bar{c}(2^3P_0)$  although the contribution of the molecular  $DD$  one is also significant. This structure provide a possible explanation for the unusual decay mode  $\omega J/\psi$  through the rescattering  $0^{++} \rightarrow DD \rightarrow \omega J/\psi$ .

Then our calculation shows that the  $X(3915)$  and the  $Y(3940)$  may be two different resonances as measured by Belle.

### B. Negative parity sector

In the  $J^{PC} = 1^{--}$  sector one can find a similar situation to those studied in the previous sector. In fact the  $\psi(4040)$  resonance with a mass just above the  $D^* D^*$  threshold has been proposed long ago [48] as a candidate to a molecular state. To asses this possibility we have performed a coupled channel calculation including the  $c\bar{c}(3^3S_1)$  and  $c\bar{c}(2^3D_1)$  states with masses  $M = 4097.615$  MeV and  $M = 4152.715$  MeV respectively, together with the channels  $DD$ ,  $DD^*$ ,  $D^* D^*$ ,  $D_s D_s$ ,  $D_s D_s^*$  and  $D_s^* D_s^*$ . The results of the calculation are shown in Table VIII.

The first striking outcome of the calculation (see Ta-

ble VIII) is the appearance of a narrow state at  $M = 3994.6$  MeV. A state with this characteristics has been reported by BaBar [49] in the study of exclusive initial-state-radiation production of the  $DD$  system. Its experimental mass and width are respectively  $M = 3943 \pm 17 \pm 12$  MeV and  $\Gamma = 52 \pm 8 \pm 7$  MeV. This resonance has also been seen by the Belle Collaboration [50]. The second significant result is that, due to the molecular mixing, the  $c\bar{c}(2^3D_1)$  state becomes the most important component of the  $\psi(4040)$  and not the  $c\bar{c}(3^3S_1)$  as usually attributed by the naive quark model. Moreover the  $\psi(4040)$  acquires a significant  $DD^*$  (23.49%) and  $D^* D^*$  (25.81%) components while the  $\psi(4160)$  is predominantly a  $c\bar{c}(3^3S_1)$  state with small contributions ( $< 20\%$ ) of different  $DD$  and  $D_s D_s$  channels.

These new assignments have an important influence on the decay ratios of these two resonances. It is well known that the ratios of branching fractions involving these two resonances shows significant discrepancies with model predictions, specially with the  $^3P_0$  model. The predictions of our calculation are shown in Table IX together with two different quark models. As seen, the new assignment improve the overall agreement with the experimental data although some discrepancies remains.

Another consequence of the important molecular component of the  $\psi(4040)$  is that it can give rise to an enhancement of some specific decay channels like the  $\psi(4040) \rightarrow \eta J/\psi$  through the presence of  $(D^* D^*)_{S=0}$  pairs in the  $\psi(4040)$  internal structure. This mechanism has been recently proposed by Voloshin [54] as a tool to identified possible molecular structures in this states.

## IV. SUMMARY

During the years charmonium spectrum below  $DD$  threshold was a well described system in quark models. However since the last 10 years new states above this threshold has been measured with properties which can be hardly described in naive quark models. Obviously at these energies threshold effects has to be taken into account which sometimes has been referred as an unquenching of the quark model.

In this paper we address this issue focusing on the possible influence of the molecular structures on the charmonium spectrum. In the framework of a constituent quark model, we have developed a formalism which allows us to coupled  $c\bar{c}$  states with four quark molecular states below and above the different thresholds. We study the positive parity sector in the mass region of the  $X(3872)$  and the negative parity sector around masses of 4.0 GeV.

We describe the  $X(3872)$  resonance as a  $J^{PC} = 1^{++}$  mixture of neutral and charged  $DD^*$  molecular states and a less than 10% contribution of the  $c\bar{c}(2^3P_1)$  state what however is enough to describe the electromagnetic decays of the resonance. The isospin breaking showed by the data is also well explained with this configuration. Together with this resonance in the  $1^{++}$  sector appears a



$M(\text{MeV})$	$c\bar{c}3^3S_1$	$c\bar{c}2^3D_1$	$DD$	$DD^*$	$D^*\bar{D}^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$	$D_s^*\bar{D}_s^*$
$3994.6 - i11.60$	31.56	3.0	2.49	36.44	117.75	7.53	0.523	0.71
$4048.4 - i7.54$	0.92	36.15	2.99	23.49	25.81	8.86	0.924	0.85
$4123.9 - i71.11$	59.01	0.98	2.13	6.84	19.119	0.75	3.37	7.73

Table VIII: Mass (in MeV) and channel probabilities (in %) for the  $J^{PC} = 1^{--}$  sector in the 4.0 GeV region.

$ratio$	$Measurements$	$^3P_0$ [51]	$C^3$ [52]	This work
$\mathcal{B.R.}(\psi(4040) \rightarrow DD)/\mathcal{B.R.}(\psi(4040) \rightarrow DD^*)$	$0.24 \pm 0.05 \pm 0.12$	0.003	0.0003	0.07
$\mathcal{B.R.}(\psi(4040) \rightarrow D^*\bar{D}^*)/\mathcal{B.R.}(\psi(4040) \rightarrow D\bar{D}^*)$	$0.18 \pm 0.14 \pm 0.03$	1.0	1.0	0.61
$\mathcal{B.R.}(\psi(4160) \rightarrow D\bar{D})/\mathcal{B.R.}(\psi(4160) \rightarrow D^*\bar{D}^*)$	$0.02 \pm 0.03 \pm 0.02$	0.46	0.008	0.0.5
$\mathcal{B.R.}(\psi(4160) \rightarrow D\bar{D}^*)/\mathcal{B.R.}(\psi(4160) \rightarrow D^*\bar{D}^*)$	$0.34 \pm 0.14 \pm 0.05$	0.011	0.16	0.08

Table IX: Ratios of branching fractions for the two  $\psi$  resonances. Experimental data are from ref [53].

second state whose properties are compatible with those of the  $X(3940)$  state.

We found two resonances with  $J^{PC} = 0^{++}$  quantum numbers. The first one, with important  $c\bar{c}$  and  $DD$  components, may be identified with the  $X(3915)$ . We found also a second broad resonance in the mass region  $M = 3940$  MeV which could be assign to the signal seen by the Belle Collaboration and which for sometime was called  $Y(3940)$ , although recently, probably due to the lack of quantum numbers to accommodate this resonance, was subsummed under the name  $X(3915)$ .

Concerning the negative parity sector, we confirm the old suggestion of De Rújula *et al.* [48] that the  $\psi(4040)$  resonance is mostly a molecular state. Moreover the coupling with these molecular structures change the  $c\bar{c}$  quantum numbers of the  $\psi(4160)$  which acquires an important  $^3S_1$  component, contrary to the usual hypothesis

that is a  $^3D_1$  state. This new assignment has important consequences on the decay branching ratios. Finally the molecular components of these two resonances open the possibility of enhance the probability of new decay channels to detectable levels which deserves further studies.

### Acknowledgments

This work has been partially funded by Ministerio de Ciencia y Tecnología under Contract Nos. FPA2010-21750-C02-02, by the European Community-Research Infrastructure Integrating Activity 'Study of Strongly Interacting Matter' (HadronPhysics2 Grant No. 227431) and by the Spanish Ingenio-Consolider 2010 Program CPAN (CSD2007-00042).

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